



LECTURE 32

Hashing and Efficient Data Access

CSC211 · Algorithms and Data Structures

Agenda

1 The need for $O(1)$ data access, and the hashing concept

2 Hash tables, hash functions, and what makes one "good"

3 Collisions, and open vs. closed hashing

4 Separate chaining, linear probing, and double hashing

5 Rehashing, worked through with real before/after table sizes

6 Chaining vs. probing performance, and real applications of hashing

Hashing Concept

- A BST (Unit 5) gets search to $O(\log n)$. Hashing asks a bolder question: what if you could compute exactly where a value belongs, and skip comparisons entirely?



```
table[5] = ("apple", value)
```

- A hash table stores key-value pairs, using a hash function to convert a key directly into an array index — the same $O(1)$ array indexing from Lecture 4, now applied to arbitrary keys.

Hash Function

hash_table_chaining.cpp

```
int hashFunction(const string& key, int tableSize) {  
    int sum = 0;  
    for (char c : key) sum += c;  
    return sum % tableSize;  
}
```

- Sums ASCII values of every character, then takes modulo the table size.
- A simple hash — easy to understand, but poor in practice.
- Notice: "cherry", "fig", and "grape" all hash to index 2 — a COLLISION.

hash("apple", 7) = 5

hash("cherry", 7) = 2

hash("fig", 7) = 2

hash("grape", 7) = 2

Properties of a Good Hash Function



Deterministic

The same key always produces the same index.



Fast to compute

The whole point is speed — a slow hash defeats the purpose.



Uniform distribution

Keys should spread evenly, not cluster in a few indices.

A collision happens when two different keys hash to the same index.

This is COMMON, not rare — "cherry"/"fig"/"grape" collided at table size 7. A second, independent check confirms it isn't a fluke: "lemon"/"melon"/"cocoa" all hash to 0 at table size 11, despite sharing no letters and different lengths.

TABLE SIZE 7: INSERT APPLE, CHERRY, FIG, GRAPE

A Collision, Visualized

0	(empty)	
1	(empty)	
2	cherry → fig → grape	← 3-entry chain (collision)
3	(empty)	
4	(empty)	
5	apple	
6	(empty)	

Five of seven buckets sit empty while bucket 2 holds a three-entry chain — exactly the "clustering" a good hash function tries to avoid.

Open Hashing: Separate Chaining

`hash_table_chaining_full.cpp`

```
void insert(const string& key, int value) {
    int index = hashFunction(key);
    table[index].push_back({key, value});
}

bool search(const string& key, int& valueOut) {
    int index = hashFunction(key);
    for (auto& p : table[index])
        if (p.first == key) { valueOut = p.second; return true; }
    return false;
}
```

- Every key that hashes to an index gets appended to that index's list.
- `search()` still finds a match correctly — it just checks each entry in the bucket instead of one direct step.
- Found "fig": 40 (correctly found despite the collision).

Closed Hashing: Linear Probing

- On a collision, checks index+1, then index+2, wrapping around, until an empty slot is found.



"cherry", "fig", and "grape" all "want" index 2 — instead of a list, linear probing physically walks forward: "fig" lands at 3 (1 probe), "grape" at 4 (2 probes).

Linear Probing — Code, and Beyond

`hash_table_probing.cpp`

```
int index = hashFunction(key);
while (occupied[index]) {
    index = (index + 1) % tableSize; // wrap around
}
keys[index] = key; occupied[index] = true;
```

- Linear probing's weakness: CLUSTERING — once keys land near each other, new collisions land there too, more often.
- Quadratic probing and double hashing both address clustering by spreading probes out more.

Quadratic probing

Checks $\text{index}+1^2$, $+2^2$, $+3^2$ — spreads probes out faster.

Double hashing

A second hash function computes the probe step size itself.

GROWING THE TABLE BEFORE LOAD FACTOR GETS TOO HIGH

Rehashing

- When load factor (entries ÷ table size) exceeds a threshold, a new, larger table is built and every key re-inserted.

Before rehash — size 4, load factor 1.00

0	(empty)
1	cherry
2	apple→fig
3	grape

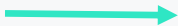


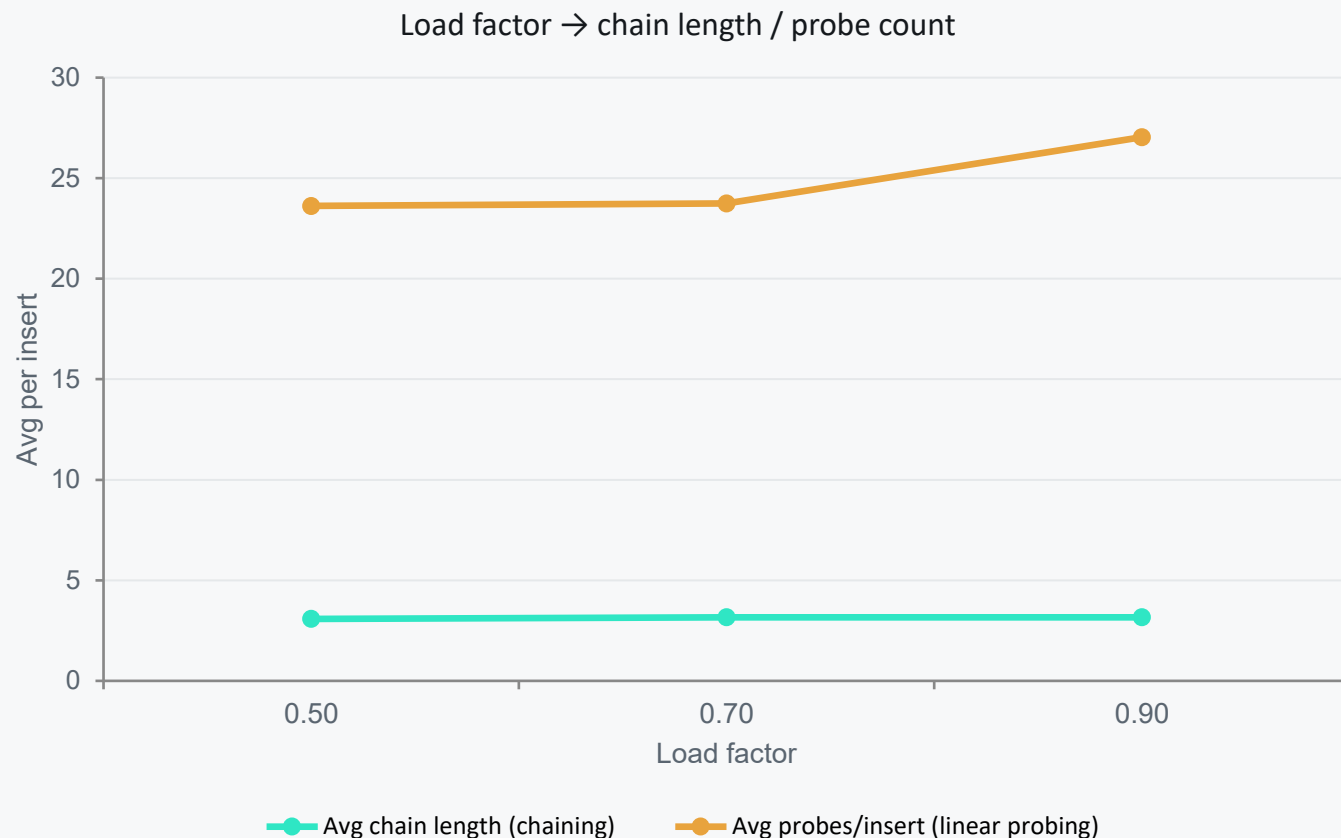
table size
4 → 8

After rehash — size 8

0	(empty)
1	(empty)
2	apple→mango
3	lemon
4	kiwi
5	cherry
6	fig
7	grape

Every key's index is recomputed from scratch at the new size — "fig" moved to bucket 6 alone, while "apple" stayed at bucket 2 (later joined by "mango"). Cost amortizes to O(1) per insertion.

Chaining vs. Probing: Measured Data



- Chaining's average chain length stays a modest ~ 3 across all load factors.
- Linear probing's average probe count balloons past 20 and keeps climbing.
- Probing is far more sensitive to a poor hash function than chaining is.

WHERE THIS SHOWS UP EVERY DAY

Applications of Hashing

1

Dictionaries & maps

Python dict, JS objects, C++ unordered_map.

2

Caching

Browser cache, CPU cache, CDN edge cache — "do I already have this?" in $O(1)$.

3

Password storage

Passwords are hashed so the original is never stored in plain text.

4

Duplicate detection

Checking if a value has been seen before, $O(1)$ per check.



CLOSING

Key Takeaways

- Hashing computes an index directly from a key — $O(1)$ average-case access, at the cost of giving up ordering.
- A collision — two keys hashing to the same index — is common, not rare; every real hash table needs a resolution strategy.
- Separate chaining stores a list per index; linear probing keeps every entry in the array, searching forward for empty slots.
- Rehashing grows the table and re-inserts everything once the load factor gets too high, keeping $O(1)$ average performance.
- Chaining tolerates a weak hash function far better than probing does — measured data showed chain length ~ 3 vs. probes climbing past 20.



COURSE COMPLETE

You've Reached the End of CSC211

Algorithms and Data Structures

- You started with one question: "how should data be organized?" — and built, in real compiled C++, every major answer to it.
- Linear structures that trade access speed against insertion cost; trees that turn search into a logarithmic walk; graphs that model any network of relationships; and now hashing, which sidesteps search entirely with direct computation.
- Every structure here reappears constantly ahead: databases index tables with trees and hash tables, operating systems schedule with queues, compilers parse with trees and stacks.

The goal was never to memorize implementations — it was to build the habit of asking: what does this problem need to do quickly, and which structure gives me that?