



LECTURE 29

Searching Algorithms

CSC211 · Algorithms and Data Structures

Agenda

- 1 The searching problem, formally
- 2 Linear search, and its complexity
- 3 Sentinel linear search — a small optimization on the same idea
- 4 Binary search — iterative and recursive — and its complexity
- 5 Binary search variants: first and last occurrence of a duplicate value
- 6 Common pitfalls, and a direct comparison of the two strategies

The Searching Problem

- Given n elements and a target value, searching answers: does the target exist, and if so, at what position?
- Every searching algorithm trades off how much work is needed per search against how much preparation (like sorting) is needed beforehand.
- This lecture compares the two fundamental strategies precisely, and shows why one needs the data sorted first.

42	17	89	3	56
----	----	----	---	----

`target = 89`

`found at index 2`

?

Present? Where?

The two questions every search answers.

STRATEGY 1

Linear Search

- Checks every element in order until it finds the target or exhausts the collection — no assumptions about data order required.

Searching for 56:

42	17	89	3	56	71	8
0	1	2	3	4	5	6

checked: 42, 17, 89, 3 → then found 56 at index 4 (5 comparisons)

$O(n)$

worst case: target absent or last

Best case (target first): $O(1)$ • Worst case (last / absent): $O(n)$

Linear Search — Code

```
searching_algorithms.cpp  
  
int linearSearch(const vector<int>& data, int target)  
{  
    for (int i = 0; i < data.size(); i++) {  
        if (data[i] == target) return i;  
    }  
    return -1;  
}
```

- One comparison per element, checked in order.
- Returns the index on the first match.
- Returns -1 if the loop finishes without finding the target.

Linear search for 56 in unsorted data:

index 4

Linear search for 99 (absent):

index -1

A SMALL OPTIMIZATION

Sentinel Linear Search

- The plain loop does TWO comparisons per iteration: the bounds check ($i < n$) and the actual value check.
- Sentinel search plants the target at the very last slot, guaranteeing the loop always finds a match — only ONE comparison per iteration remains.
- Still $O(n)$ worst case — same growth rate — but a real, measurable constant-factor speedup.

Array with target 8 (the real last element) as sentinel-safe search:

42	17	89	3	56	71	8
----	----	----	---	----	----	---

sentinel slot

```
sentinel_linear_search.cpp
```

```
int last = data[n - 1];  
data[n - 1] = target;    // plant sentinel
```

```
int i = 0;  
while (data[i] != target) i++;  
// only ONE comparison / iteration
```

$\frac{1}{2}$

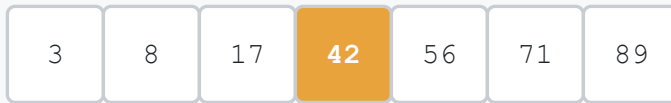
the comparison count per iteration

STRATEGY 2 — REQUIRES SORTED DATA

Binary Search

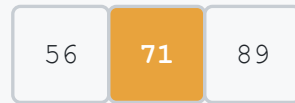
- Requires sorted data — in exchange, eliminates HALF the remaining search space with every comparison.

Step 1: full range



mid=42. $56 > 42 \rightarrow$ search RIGHT half

Step 2: right half



mid=71. $56 < 71 \rightarrow$ search LEFT half

Step 3: found



mid=56. Found! (3 comparisons)

3 comparisons found 56 in a 7-element array — linear search would have needed 5 (checking 3, 8, 17, 42 first).

Binary Search — Code

searching_algorithms.cpp

```
int low = 0, high = sortedData.size() - 1;
while (low <= high) {
    int mid = low + (high - low) / 2;
    if (sortedData[mid] == target) return mid;
    else if (sortedData[mid] < target) low = mid + 1;
    else high = mid - 1;
}
return -1;
```

- `'low + (high-low)/2'` avoids integer overflow that `'(low+high)/2'` risks on large arrays.
- Each comparison halves the search space by moving low or high.
- A recursive version follows the exact same logic.

$O(1)$

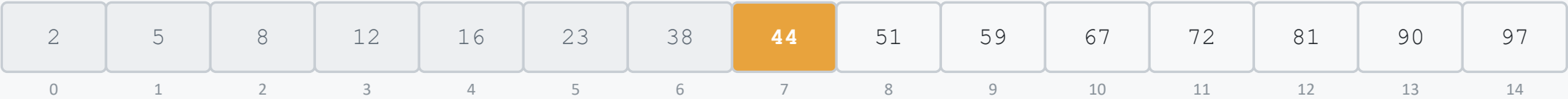
best case: target is the middle element

$O(\log n)$

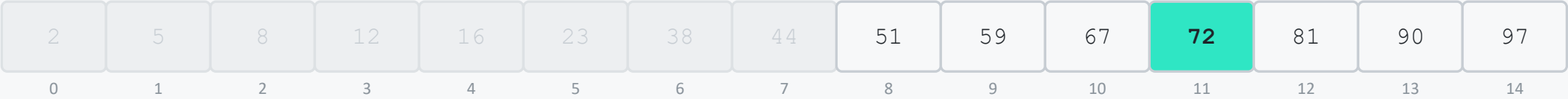
worst case: target at an edge, or absent

Tracing Binary Search on 15 Elements

Step 1: low=0, high=14, mid=7 → 44. 72 > 44 → search right half



Step 2: low=8, high=14, mid=11 → 72. 72 == 72 → FOUND at index 11

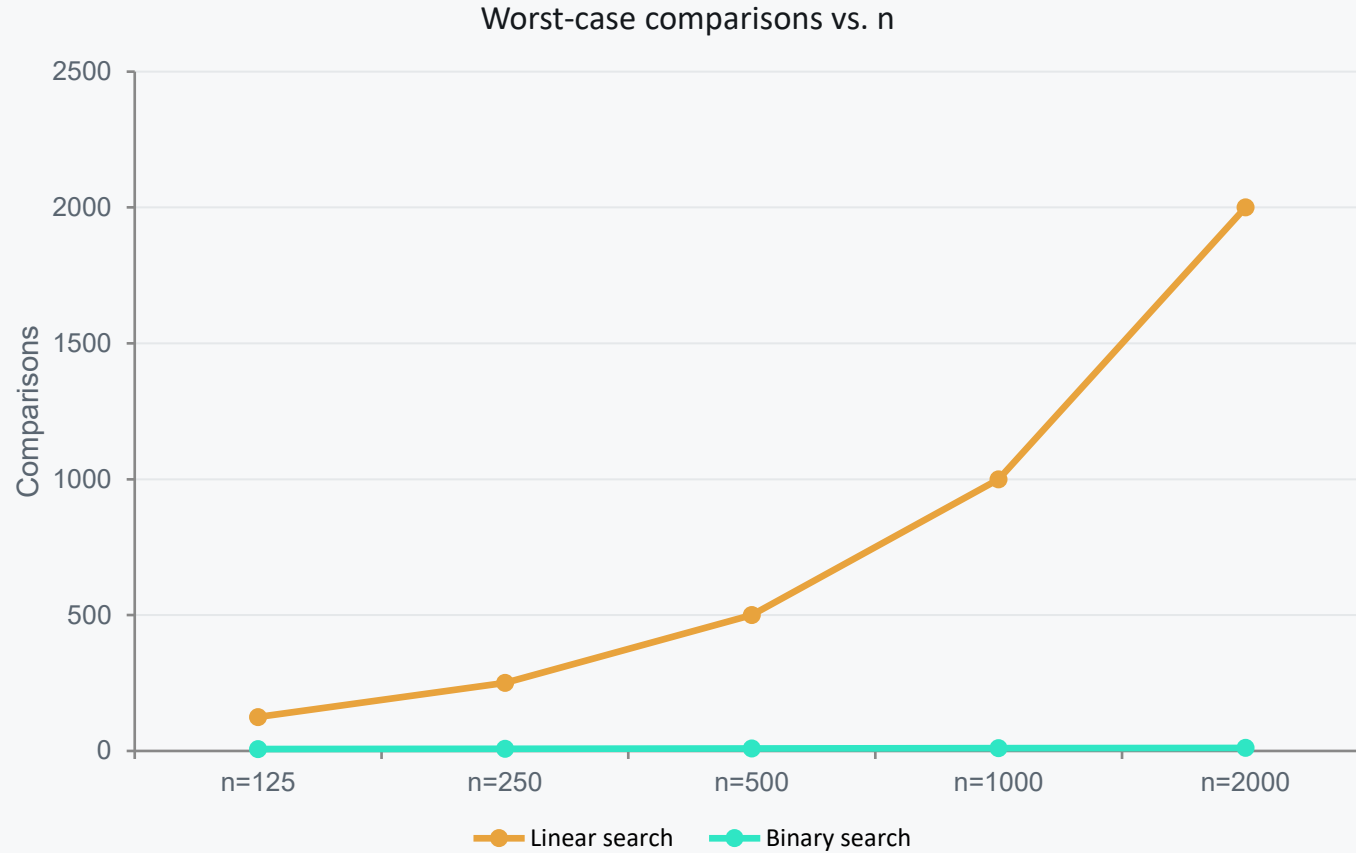


Two comparisons located the target among 15 elements. The gap only widens as n grows.

2

comparisons vs. 12 for linear search

Measuring the Advantage



2000 vs 11

comparisons: linear vs. binary at n=2000

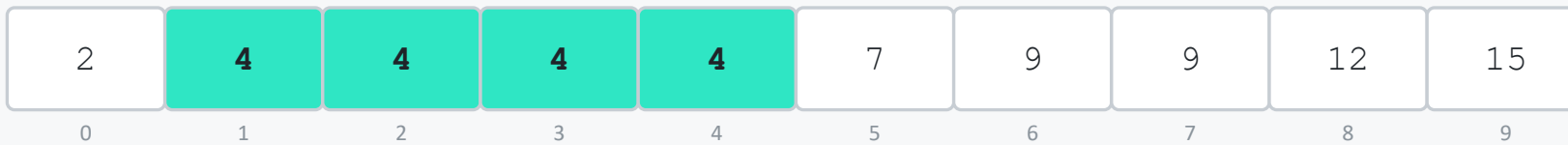
- Linear search's worst case scales linearly with n.
- Binary search needs only $\text{ceil}(\log_2 n)$ comparisons — one more per doubling of n.
- This is exactly why $O(\log n)$ is called "logarithmic."

First and Last Occurrence

- On finding a match, don't stop — record it, then keep narrowing toward an earlier (or later) one.

first →

findFirst(4) narrows LEFT after each match → index 1 findLast(4) narrows RIGHT → index 4



findFirst — the ONE line that differs

```
if (sortedData[mid] == target) {  
    result = mid;  
    high = mid - 1; // keep searching LEFT  
}
```

findLast — the ONE line that differs

```
if (sortedData[mid] == target) {  
    result = mid;  
    low = mid + 1; // keep searching RIGHT  
}
```

Common Pitfalls in Binary Search



Integer overflow

$\text{mid} = (\text{low} + \text{high}) / 2$ can overflow on large arrays; use $\text{low} + (\text{high} - \text{low}) / 2$ instead.



Off-by-one loop condition

`while (low < high)` silently skips the single-element case; use `low <= high`.



Unenforced sorted precondition

Binary search doesn't fail loudly on unsorted data — it just returns wrong answers.



Infinite loops

`low = mid` instead of `low = mid + 1` can leave the search space unchanged forever.

Linear vs. Binary Search

	Linear Search	Binary Search
Requires sorted data?	No	Yes
Worst-case complexity	$O(n)$	$O(\log n)$
Works on a linked list?	Yes (sequential access)	Poorly (needs random access)

- A one-time search of unsorted data rarely justifies sorting first (which itself costs $O(n \log n)$) plus a binary search.
- Searching the same data repeatedly: sorting once and reusing binary search every time wins decisively.



CLOSING

Key Takeaways

- Linear search makes no assumptions about order ($O(n)$ worst case); sentinel search halves the per-iteration comparisons.
- Binary search needs sorted, random-access data in exchange for $O(\log n)$ — confirmed with a 15-element trace and a 2,000-element count.
- `findFirst` / `findLast` generalize binary search to locate the boundaries of a run of duplicates.
- Short code hides real pitfalls: midpoint overflow, off-by-one loops, unenforced sort order, infinite loops.
- The right choice depends on how the data is organized and how often it's searched.